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Implementation of Logarithmic Calculation Using Integer Arithmetic on LUA without libraries

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Common practice, for visually displaying data with a large range of quantities, is to use a logarithmic scale, as linear representations can often hide or mask some important structures and on the other hand emphasize some less significant ones. However, a logarithmic scale may appear less then optimal, or at least hard to implement, when using micro-controllers and other processors that allow only integer arithmetic.

An option, especially with smaller data range, is to construct a comparison table that maps an integer value to the nearest logarithmic value, again represented by an integer. Data range in the test example was between 1 and 1024, which resulted in the total of 31 comparisons groups. The computational time ranged from 190us to 430us depending on the magnitude of input integer.

An alternative approach is possible, exploiting basic arithmetic properties of logarithms, with not much larger computational time. From mathematical prospective, a logarithmic integer calculation function is constructed using a base 10 logarithmic map:

$$x \mapsto 10 \log_{10} x. \tag{1}$$

To make an accurate approximation to the nearest integer, transformation to the natural logarithm is made:

$$10 \log_{10} x = 10 \frac{\ln x}{\ln 10}. \tag{2}$$

First, some properties of a natural logarithm are observed:

1. $\ln xy = \ln x + \ln y$,
2. $\ln e = 1$,
3. for $x \in (0, 1]$ ln can be approximated using Taylor series expansion

$$\ln x = \sum_{1}^{\infty} \frac{(-1)^{n+1}}{n} (x-1)^n. \tag{3}$$

The idea is to divide x by e until we are left with a number smaller then 1. Then 1 is added to the result for each division. The Taylor series approximation is used to correct the result. As only integer division is possible, where the remainder is discarded, each integer is multiplied by 100, so

we can still keep track of the remainder. Approximations are used for $e \sim 2.72$ and $\ln 10 \sim 2.30$ to two decimal places. Note that larger scaling and more accurate approximations can be used for more accuracy.

Approximation is done in two steps. The code provided below is in lua programming language.

1. First, the division by e takes place

```
E=272
ln10=230
ans=0
res=0

while x > 0 do
        res =res*100/E
        a1 = x*10000/E
        x = a1/100;
        res = res+a1-x*100;
        ans=ans+1;
        if res>100 then
                x=x+1;
                res=res-100;
        end
end
```

   As we can see, the division is repeated until number less then 1 is left. Two digit remainder is recorded and adjusted at each step.

2. In the second step, the result is adjusted using Taylor series approximation, then converted back to the log in base 10.

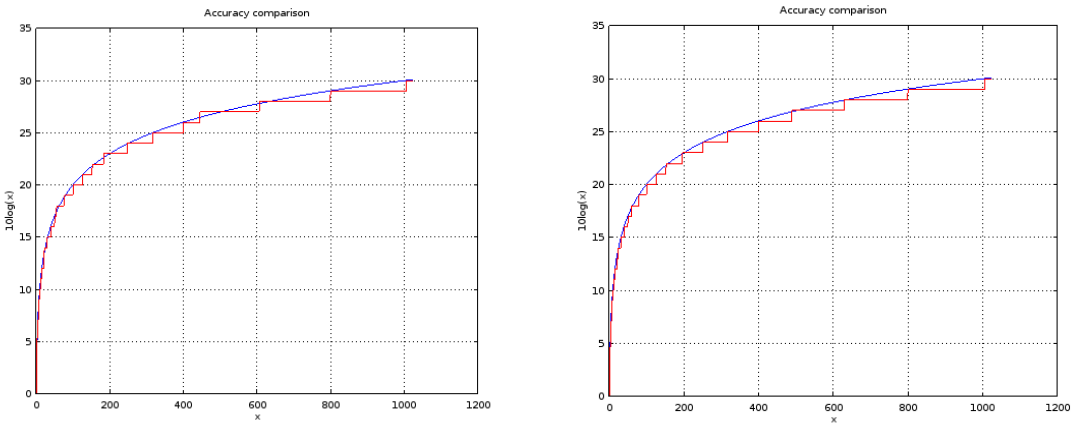
```
res = res-100 - (100-res)*(100-res)/200
ans = (ans*100+res)/100
return ans*1000/ln10
```

The computation time is between 221us and 492us, depending on the magnitude of the input integer. Thus, the rigid comparison table can be replaced by much neater solution with comparable execution time. The solution becomes even more convenient when dealing with data of larger magnitude.

The above approximation should yield results with the integer accuracy in most cases. In particular, value of $10 \log_{10} x$ is rounded to the integer, discarding the residue. Comparison between the actual value and approximation is displayed in the Figure1a below. Such accuracy should be satisfactory for most visualisations, but may be an less then optimal if data is spaced closely together. In such cases an alternative method should be considered.

It is possible to achieve slightly better accuracy by using larger scaling of numbers, together with more accurate approximations for constants. However, for more evident improvement additional terms in the Taylor series should be considered. In the Figure below, we can see the difference

between using two and three terms of expansion. It is advised to examine the pay-off between the computation time and accuracy first, before implementing any of the mentioned improvements.



(a) Two terms of Taylor series.

(b) Three terms of Taylor series.

Slika 1: Comparison between the exact value of $10 \log_{10} x$ and approximation.